NASA TECHNICAL TRANSLATION NASA TT F-11,642

NASA TT F-11,642

CALCULATION OF A HYPERSONIC VISCOUS GAS FLOW AROUND A SPHERE NEAR THE STAGNATION LINE

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Translation of "Raschet giperzvukovogo vyazkogo obtekaniya sfery vblizi linii tormozheniya
In: Vychistlitel'yye Metody i Programmirovaniye; Chislennyye Metody v Gazovoy Dinamike. Sbornik Rabot Vychislitel'nogo Tsentra Moskovskogo Universiteta, No. 7, pp. 68-82, 1967.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 AUGUST 1968

CALCULATION OF A HYPERSONIC VISCOUS GAS FLOW AROUND A SPHERE NEAR THE STAGNATION LINE

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ABSTRACT: A steady hypersonic flow of a viscous heat-conducting compressible gas is considered on the basis of Navier-Stokes equations written in spherical polar coordinates. It is assumed that the gas is perfect and monatomic and that its volume viscosity is equal to zero, while the viscosity and heat-conductivity coefficients are proportional to the square root of the absolute temperature T. An approximate local self-similar solution is used, which makes it possible to obtain flow characteristics in the stagnation line region. A nonstationary system obtained by using an explicit difference scheme is integrated numerically in the region from the undisturbed flow down to the body and passing through the shock-wave zone. As a result, the structures of the shock layer and shock wave are determined. The results are presented of a numerical calculation of flow past a thermally insulated sphere at $M_\infty = 10$, $\gamma = 5/3$, $Pr = 3/4$, with $Re = 50$, 100, 500, and 1000 characterizing the transition from a rarefied to a dense gas with gradual formation of a thin shock wave. Diagrams of pressure, velocity, and density distributions are included. It is stated that the computation procedure outlined here may be used for calculations of plane as well as axisymmetrical bodies with nose section of nearly cyclindrical or spherical shape and with thermally insulated or cool surface.

This paper uses the "developed flow" method for calculating steady state /68*
hypersonic flow of viscous gas around a sphere. Use is made of an approximate local self-similar solution, which makes it possible to obtain flow characteristics near the stagnation line. An explicit difference scheme is used to integrate the nonstationary system in a region extending from the undisturbed flow and up to the body with transition through a shock-wave zone; no assumptions are made here regarding the behavior of the flow in this region. The structure of the shock layer ahead of the sphere, including the shock-wave structure, is determined as a result.

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*Numbers in the margin indicate pagination in the foreign text.

Examples of calculations are presented for flow around a thermally-insulated sphere at $M_\infty = 10$, and Reynolds numbers 50, 100, 500 and 1000.

1. Statement of the Problem

We are considering the calculation of steady-state flow around a sphere in a homogeneous hypersonic flow of viscous, thermally-conducting, compressible gas. The starting equations for the problem are Navier-Stokes equations written in spherical polar coordinates (see, for example [1]). It is assumed that the gas is perfect and monatomic, for which the volume [bulk] viscosity is zero, while the viscosity μ and the thermal conductivity k are proportional to the square root of T, (the absolute temperature). The ratio of specific heats $\gamma = c_p/c_v$ and the Prandtl number $Pr = c_p\mu/k$ are assumed to be constant. As a result we have a closed system of equations with respect to the following desired functions: u, v which are the radial and tangential components of the velocity, ρ - the density, p - the pressure, $h = c_p T$ - the enthalpy and μ - the viscosity.

The boundary conditions of the problem are those prevailing on the surface of a sphere at $r = r_u$ (Fig. 1) and the conditions in the undisturbed incident flow /69

$$M_\infty,\ Re_\infty,\ U_\infty,\ \rho_\infty,\ p_\infty.$$

We assume that the conditions on the sphere surface assure flow adhesion to that surface:

$$u(r_w, \theta) = 0,\ v(r_w, \theta) = 0 \tag{1}$$

We also assume that the sphere is thermally insulated:

$$\frac{\partial}{\partial r} h(r_w, \theta) = 0. \tag{2}$$

For the case of very small Reynolds numbers (when the gas is sufficiently rarefied but the continuous-medium hypothesis is still valid) conditions (1) and (2) must be replaced by slip and temperature jump conditions.

Since it is assumed that the flow far from the body is homogeneous and undisturbed, we have the following conditions at $r \to +\infty$

$$\begin{gathered} u(r, 0) = -U_\infty \cos\theta,\ v(r, \theta) = U_\infty \sin\theta, \\ p(r, \theta) = p_\infty,\ \rho(r, \theta) = \rho_\infty. \end{gathered} \tag{3}$$

The boundary-value problem for complete Navier-Stokes equations stated in this manner can be solved numerically only with extreme difficulty. We shall follow an approximate method for solving this problem suggested by Probstein and Kemp [1]. According to this method the Navier-Stokes equations in supersonic flow in the shock layer ahead of a blunt body can be simplified ([2], page 67) and then, using the concept of local self-similarity, it can be reduced to a system of ordinary differential equations. The solution of the problem thus obtained will be valid only in the vicinity of the stagnation line $\theta = 0$, which passes only through the forward stagnation point on the body.

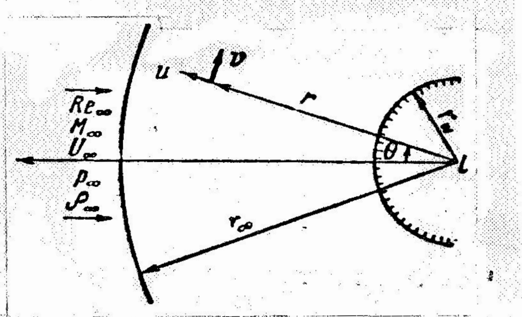

Figure 1.

The following form of the local self-similar solution was used when changing to ordinary differential equations /70

$$u(r, \theta) = u_0(r)\cos\theta, v(r, \theta) = v_0(r)\sin\theta, \rho(r, \theta) = \rho_0(r),$$
$$h(r, 0) = h_0(r)\cos^2\theta, \mu(r, \theta) = \mu_0(r)\cos\theta,$$
$$p(r, \theta) = p_0(r)\cos^2\theta + p_2(r)\sin^2\theta. \tag{4}$$

According to [4], this form of solution in the series expansion of the flow parameters near axis with $\theta = 0$ in powers of $\sin\theta$ corresponds to the so-called "first cutoff."

The system of ordinary differential equations derived in [2] has the form

$$\frac{d}{d\bar{r}}(\bar{\rho}\bar{u}) + 2\bar{\rho}(\bar{u} + \bar{v}) = 0,$$
$$\bar{\rho}\bar{u}\frac{d\bar{u}}{d\bar{r}} = -\frac{d\bar{p}}{d\bar{r}} + \frac{4}{3Re}\cdot\frac{d}{d\bar{r}}\left(\bar{\mu}\frac{d\bar{u}}{d\bar{r}}\right) + \frac{2}{3Re}\cdot\bar{\mu}\cdot\frac{d\bar{v}}{d\bar{r}} - \frac{4}{3Re}\bar{v}\frac{d\bar{\mu}}{d\bar{r}},$$
$$\bar{\rho}\bar{v}\frac{d\bar{v}}{d\bar{r}} = \frac{1}{Re}\cdot\frac{d}{d\bar{r}}\left(\bar{\mu}\frac{d\bar{v}}{d\bar{r}}\right) - 2(\bar{p}_2 - \bar{p}) - \bar{\rho}\bar{v}(\bar{u} + \bar{v}),$$
$$\frac{d\bar{p}_2}{d\bar{r}} = \bar{\rho}\bar{v}(\bar{u} + \bar{v}). \tag{5}$$

System (5) is completed by writing the necessary relationships for the pressure, enthalpy and viscosity

$$\bar{p} = \frac{\gamma - 1}{\gamma}\bar{\rho}\bar{h}, \bar{h} = A - \frac{\bar{u}^2}{2}, \bar{\mu} = \left(\frac{\bar{h}}{A}\right)^{\frac{1}{2}}, \left(A = \frac{1}{2} + \frac{1}{(\gamma - 1)M_\infty^2}\right). \tag{6}$$

The system described by Eqs. (5) and (6) is written in dimensionless variables, which are defined as follows

$$\bar{r}=\frac{r}{r_w},\ \bar{u}=\frac{u_0}{U_\infty},\ \bar{v}=\frac{v_0}{U_\infty},\ \bar{p}=\frac{p_0}{\rho_w U_\infty^2},\ \bar{P}_2=\frac{P_2}{\rho_w U_\infty^2},$$
$$\bar{\rho}=\frac{\rho_0}{\rho_w},\ \bar{h}=\frac{h_0}{h_w},\ \bar{\mu}=\frac{\mu_0}{\mu_w};$$
$$\mathrm{Re}_\infty=\frac{r_w\rho_\infty U_\infty}{\mu_w},\ \mathrm{Re}=\frac{r_w\rho_w U_\infty}{\mu_w},\ \mathrm{Pr}=\frac{c_p\mu_w}{k_w}. \qquad (7)$$

It should be noted that the energy equation in Eqs. (5) and (6) is replaced by Becker's integral $\bar{h}+\bar{u}^2/2=A$, which is applicable when $\mathrm{Pr}=3/4$ and only for thermally insulated surfaces of the body. /71

System (5)-(6) will be solved numerically over a finite interval with respect to variable r, assuming that the disturbances produced by the body do not in practice extend past some specified distance r_∞ from the center of the sphere. This is equivalent to postulating the possibility of applying conditions prevailing at infinity to a sphere with radius r_∞.

The dimensionless boundary conditions (1)-(3) have the form

$$\bar{u}=-1,\ \bar{v}=1,\ \bar{p}=\bar{p}_2=\bar{p}_\infty,\ \bar{\rho}=\bar{\rho}_\infty \text{ when } \bar{r}=\bar{r}_\infty, \qquad (8)$$

$$\bar{u}=\bar{v}=0,\ \frac{d\bar{h}}{d\bar{r}}=0 \text{ when } \bar{r}=\bar{r}_w=1. \qquad (9)$$

In addition, by virtue of Becker's integral and of the method used for transformation to dimensionless quantities, we obtain at the surface of the body

$$\bar{\rho}(1)=1,\ \bar{\mu}(1)=1,\ \bar{h}(1)=A. \qquad (10)$$

Since the velocity field at infinite distance from the body is homogeneous and there are no rates of deformation of the flow to be considered, we can use the formulas for an ideal inviscid gas for the undisturbed incident flow:

$$\bar{h}_\infty=\frac{1}{(\gamma-1)M_\infty^2},\ \bar{\mu}_\infty=\left(\frac{\bar{h}_\infty}{A}\right)^{\frac{1}{2}},\ \bar{\rho}_\infty=\frac{\mathrm{Re}_\infty}{\mathrm{Re}}\bar{\mu}_\infty,\ \bar{p}_\infty=\frac{\bar{\rho}_\infty}{\gamma M_\infty^2}. \qquad (11)$$

As shown by calculations performed by Levinsky and Yoshihara [2], the problem cannot be solved numerically without great difficulty even if simplified as above. Hence additional simplifications are introduced by a number of authors. Thus, [1] develops the method of a "viscous" shock layer in which an infinitesimally thin shock wave serves as the outer boundary, with Hugoniot relationships prevailing on this boundary; in addition, it is assumed that the flow has a constant density. Reference [4] considers the third approximation of the boundary-layer theory, where it is assumed that viscosity is appreciable only in the shock-wave region and in the boundary layer at the body, and that these two thin layers are separated by a region of almost inviscid flow.

We should emphasize that the analysis performed in [1]-[4] is based on the assumption that the solutions are locally self-similar. It is shown in part I of [4] that this assumption makes it possible to determine the flow variables along the stagnation line with sufficiently high accuracy (it should be noted, however, that this was checked only for the case of a "viscous" shock layer).

In this paper, as in [2] we integrate directly while assuming transition through the shock-wave zone. However, we make no assumptions as to the behavior of the flow between the undisturbed flow and the body. The numerical computation presented here makes it possible to overcome difficulties encountered in the Levinsky-Yoshihara paper. /72

2. The Numerical Solution

The problem formulated in Sec. 1 is a two-point boundary-value problem for a nonlinear system of ordinary differential equations. It was solved in [2] by direct integration assuming transition through a shock wave. However, the solution involved great difficulties due to instability of the integration process, which was performed only in one direction, i.e., from the undisturbed flow to the body, or conversely. Hence it was necessary to integrate Eqs. (5)-(6) simultaneously in two directions, i.e., along the flow and against it, starting from both ends, i.e., from the undisturbed flow and from the body. Then both solutions had to be joined in some point between the body and the shock wave by selecting suitable initial conditions at both of the integration origins.

Paper [4] points out the cause of the instability operating during integration in the direction of the flow: the differential equations which describe the shock-wave structure have a singular point (a saddle point) directly behind the wave. It was noted that this disappears with a reduction in Re (for example, when $Re = 10$), at which point integration along the flow up to the surface of the body becomes practically possible. When the integration origin was at the body, an instability also appeared, this time at the approach to the shock wave. The selection of the suitable initial conditions at both integration origins and the subsequent joining of solutions which agree with each other is highly laborious and obviously affects the accuracy. We also found [5] this on repeating one of the calculations ($Re = 100$) presented in [2]: one must specify very precisely the initial data in order to achieve even a moderate accuracy in coupling the solutions.

Now we are suggesting another method for solving this boundary-value problem; this method does not require the coupling of solutions and laborious selection of missing initial data. We add to the differential equations of system (5)-(6) terms with derivatives of u, v, ρ and p_2, with respect to time t (this time /73 is not the same as the physical time in the unsteady Navier-Stokes system) and we solve the resulting unsteady-state system thus obtained by a difference method starting from certain initial conditions at t = 0 and with boundary conditions (8)-(10) which are definitely fixed in time. If there exists a single steady-state solution of the unsteady-state problem thus obtained, then we should find it during the solution as $l \to \infty$. The steady-state flow parameters will then be obtained as a result of "developed flow."

The unsteady-state system corresponding to (5)-(6) has the form (the bars over dimensionless quantities are now dropped)

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + 2\rho(u+v) = 0, \tag{12}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) = \frac{4}{3\,\mathrm{Re}}\cdot\frac{\partial}{\partial r}\left(\mu\frac{\partial u}{\partial r}\right) + \frac{2}{3\,\mathrm{Re}}\mu\frac{\partial v}{\partial r} - \frac{4}{3\,\mathrm{Re}}v\frac{\partial \mu}{\partial r} - \frac{\partial p}{\partial r}, \tag{13}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r}\right) = \frac{1}{\mathrm{Re}}\cdot\frac{\partial}{\partial r}\left(\mu\frac{\partial v}{\partial r}\right) - 2(p_2 - p) - v(u+v), \tag{14}$$

$$\frac{\partial p_2}{\partial t} - \frac{\partial p_2}{\partial r} - \rho v(u+v) = 0, \tag{15}$$

$$h = A - \frac{u^2}{2}, \tag{16}$$

$$\mu = \left(\frac{h}{A}\right)^{\frac{1}{2}}. \tag{17}$$

$$p = \frac{\gamma - 1}{\gamma}\rho h. \tag{18}$$

This system will be solved by the method of finite differences in the region $1 \leq r \leq r_\infty$, $t > 0$. We draw on the plane of independent variables (r, t) a grid with nodes $r = 1 + ml$, $t = n\tau$; $m = 0, 1, 2, \ldots, k$, $n = 0, 1, 2, \ldots$, where $l = (r_\infty - 1)/k$. The desired functions in the nodal points are denoted by f_m^n.

To approximate the derivative in Eqs. (12)-(15), we use an explicit difference scheme suggested in [6] (henceforth,

$$\xi = \frac{\tau}{2l},\ \eta = \frac{\tau}{l^2},\ \nu = \frac{1}{Re}):$$

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$$\begin{aligned}\bar{u}_m^{n+1} &= u_m^n - \xi u_m^n (u_{m+1}^n - \bar{u}_{m-1}^{n+1}) + \\ &+ \frac{2\nu}{3\rho_m^n}\Big\{2\eta\,[0.5(\mu_{m+1}^n + \mu_m^n)(u_{m+1}^n - u_m^n) - \\ &- 0.5(\mu_m^n + \mu_{m-1}^n)(u_m^n - u_{m-1}^n)] + \xi\mu_m^n(v_{m+1}^n - \bar{v}_{m-1}^{n+1}) - \\ &- 2\eta v_m^n(\mu_{m+1}^n - \bar{\mu}_{m-1}^{n+1})\Big\} - \xi\frac{p_{m+1}^n - \bar{p}_{m-1}^{n+1}}{\rho_m^n},\end{aligned} \tag{19}$$

$$\begin{aligned}\bar{v}_m^{n+1} &= v_m^n - \xi u_m^n (v_{m+1}^n - \bar{v}_{m-1}^{n+1}) + \\ &+ \eta\frac{\nu}{\rho_m^n}[0.5(\mu_{m+1}^n + \mu_m^n)(v_{m+1}^n - v_m^n) - \\ &- 0.5(\mu_m^n + \mu_{m-1}^n)(v_m^n - v_{m-1}^n)] + \frac{2\tau}{\rho_m^n}[p - p_2 - v(u+v)]_m^n,\end{aligned} \tag{20}$$

$$\begin{aligned}\bar{\rho}_m^{n+1} &= \rho_m^n - \xi[u_m^n(\rho_{m+1}^n - \bar{\rho}_{m-1}^{n+1}) + \rho_m^n(u_{m+1}^n - \bar{u}_{m-1}^{n+1})] - \\ &- 2\tau[\rho(u+v)]_m^n,\end{aligned} \tag{21}$$

$$\bar{p}_{2.m}^{n+1} = p_{2.m}^n + \xi(p_{2.m+1}^n - \bar{p}_{2.m-1}^{n+1}) - \tau[\rho v(u+v)]_m^n. \tag{22}$$

The values of $\bar{f}_m^n$ are computed from Eqs. (19)-(22) in the direction undisturbed flow→body, using the boundary conditions (8). Since the values of ρ and p_2 are known only for the undisturbed flow (when $r = r_\infty$), then the body ($r = 1$) by virtue $u(1) = v(1) = 0$ they must be calculated from the equations $\frac{\partial \rho}{\partial t} = -\rho\frac{\partial u}{\partial r}$ and $\frac{\partial p_2}{\partial t} = \frac{\partial p_2}{\partial r}$, written in the following difference form ($m = k$)

$$\bar{\rho}_k^{n+1} = \rho_k^n - \xi\rho_k^n(u_{k-2}^n - 4u_{k-1}^n),\quad \bar{p}_{2.k}^{n} = p_{2.k} + \xi(p_{2.k-2}^n - 4p_{2.k-1}^n + 3p_{2.k}^n). \tag{23}$$

Then $\overline{\overline{f}}_m^{n+1}$ are calculated in the opposite direction from the expressions

$$\begin{aligned}\overline{\overline{u}}_m^{n+1} &= u_m^n - \xi u_m^n\left(\overline{\overline{u}}_{m+1}^{n+1} - u_{m-1}\right) + \\ &+ \frac{2\nu}{3\rho_m^n}\Big\{2\eta\,[0.5(\mu_{m+1}^n + \mu_m^n)(u_{m+1}^n - u_m^n) - \\ &- 0.5(\mu_m^n + \mu_{m-1}^n)(u_m^n - u_{m-1}^n)] + \xi\mu_m^n\left(\overline{\overline{v}}_{m+1}^{n+1} - v_{m-1}^n\right) - \\ &- 2\eta v_m^n\left(\overline{\overline{\mu}}_{m+1}^{n+1} - \mu_{m-1}^n\right)\Big\} - \xi\,\frac{\overline{\overline{p}}_{m+1}^{n+1} - p_{m-1}^n}{\rho_m^n},\end{aligned} \qquad (24)$$

$$\begin{aligned}\overline{\overline{v}}_m^{n+1} &= v_m^n - \xi u_m^n\left(\overline{\overline{v}}_{m+1}^{n+1} - v_{m-1}^n\right) + \\ &+ \eta\,\frac{\nu}{\rho_m^n}\,[0.5(\mu_{m+1}^n + \mu_m^n)(v_{m+1}^n - v_m^n) - \\ &- 0.5(\mu_m^n + \mu_{m-1}^n)(v_m^n - v_{m-1}^n)] + \frac{2\tau}{\rho_m^n}\,[p - p_2 - v(u+v)]_m^n,\end{aligned} \qquad (25)$$ /75

$$\begin{aligned}\overline{\overline{\rho}}_m^{n+1} &= \rho_m^n - \xi\left[u_m^n\left(\overline{\overline{\rho}}_{m-1}^{n+1} - \rho_{m-1}^n\right) + \rho_m^n\left(\overline{\overline{u}}_{m+1}^{n+1} - u_{m-1}^n\right)\right] - \\ &- 2\tau\,[\rho(u+v)]_m^n.\end{aligned} \qquad (26)$$

$$\overline{\overline{p}}_{2,m}^{n+1} = p_{2,m}^n + \xi\left(\overline{\overline{p}}_{2,m+1}^{n+1} - p_{2,m-1}^n\right) - \tau\,[\rho v(u+v)]_m^n. \qquad (27)$$

The values of h_m, μ_m and p_m are found from Eqs. (16)-(18). The final solution at the (n + 1)th time-dependent film is given by

$$f_m^{n+1} = 0.5\left(\overline{f}_m^{n+1} + \overline{\overline{f}}_m^{n+1}\right),\quad f = \begin{pmatrix} u \\ v \\ \rho \\ p_2 \end{pmatrix},\quad m = 1, 2, \ldots, k-1. \qquad (28)$$

This difference scheme was applied in [6] to a nodal equation

$$\frac{\partial u}{\partial t} = a\,\frac{\partial u}{\partial x} + \nu\,\frac{\partial^2 u}{\partial x^2},$$

where a and ν are constant, and ν may be small. In a solution comprising continuous derivatives up to the 4th order, the error in approximating this scheme is a quantity of the order of

$$\tau(u_{tt}^{\cdot\cdot} + u_{tx}^{\cdot\cdot}) + \frac{\tau}{l} u_t^{\cdot} + l^2 (u_{xxx}^{\cdots} + u_{xxxx}^{IV}).$$

while for steady-state conditions, i.e., when $u^{n+1} \equiv u^n$ and all the time derivatives are zero, the error will be of the order of $O(l^2)$.

The stability condition for this explicit system has the form

$$\tau \leqslant \min\left[\frac{l^2}{4\nu}, \frac{l}{|a|}\right]. \tag{29}$$

For our system it can be shown that the approximation will be of the same order also for the steady regime. Stability was studied by the Fourier method using a system of linearized equations with "frozen" coefficients, corresponding to (12)-(18), which describes the flow variables in regions where they vary slowly. The stability condition for the linearized system thus obtained is similar to condition (29) for one equation.

However, this condition was not always suitable for calculations, since the shock layer has regions with extremely variable flow parameters, and hence the spacing τ had to be made smaller than would follow from the stability condition. /76

Detailed calculations performed for this problem showed that this scheme results in the appearance of fluctuations which increase slowly with time, which, after a very large number of time-dependent layers could entirely distort the solution. To eliminate these fluctuations, smoothing was carried out in each time-dependent layer using the expression

$$(f_m^{n+1})_{\text{сгл.}} = f_m^{n+1} + \delta(f_{m-1}^{n+1} - 2f_m^{n+1} + f_{m+1}^{n+1}). \tag{30}$$

where $0 \leq \delta \leq 0.25$. Since this kind of smoothing is similar to the appearance of some fictitious viscosity, then, in order not to distort the true solution by this viscosity, we must require that

$$\varepsilon = \frac{\delta l^2}{\tau} \ll \nu = \frac{1}{Re}. \tag{31}$$

After smoothing was performed the fluctuations were eliminated and establishment of the steady-state ("developed flow") was observed.

We now show how the initial conditions are specified for $l = 0$ ($n = 0$, $m = 1, 2, \ldots, k$):

$$v_m^0 = 1 - \frac{m}{k}, \quad u_m^0 = -1 + \frac{m}{k},$$

$$\rho_m^0 = \rho_\infty + \frac{m}{k}(1 - \rho_\infty),$$

$$p_m^0 = p_\infty + \frac{m}{k}\left(\frac{\gamma - 1}{\gamma} A - p_\infty\right),$$

$$p_{2,m}^0 = p_\infty + \frac{m}{k}(p_\infty - 0.5),$$

$$h_m^0 = h_\infty + \frac{m}{k}(A - h_\infty),$$

$$\mu_m^0 = \mu_\infty + \frac{m}{k}(1 - \mu_\infty).$$

3. Results of Calculations

Calculations were performed for flow around a thermally-insulated sphere for $M_\infty = 10$, $\gamma = 5/3$, $Pr = 3/4$ for Reynolds numbers equal to 50, 100, 500 and 1000, which characterize transition from a rarefied to a dense gas with gradual formation of a thin shock wave.

Before starting calculations we specify some value of r_∞ (as a rule, not greater than two sphere radii), we set $l = (r_\infty - 1)/k$, where k is the number of points on the time-dependent layer, we take the time spacing τ in accordance to the stability condition for the linearized system. We select δ, the smoothing parameter, from Eq. (31) $\varepsilon = \alpha v$, $\alpha \sim 0{,}1 \div 0{,}5$: /77 /78

$$\delta = \alpha v \frac{\tau}{l^2}, \quad 0 < \delta \leqslant 0.25.$$

When solving the problem numerically one deals with one free parameter, i.e., the ratio $\xi = Re_\infty/Re$, on which conditions (11) depend. In varying this parameter one should try to bring about a situation whereby $\rho_u(t) \to 1$ as $t \to \infty$ (this follows from selection of the dimensionless quantities). First the "developed flow" problem is solved for $\xi = 1$, which produces some value $\rho_u \neq 1$. Then the selected ξ is corrected and the problem is solved again, but then the steady-state solution /79 for the former value of ζ is taken as the initial conditions at $t = 0$. The process of selection of ζ converges quite rapidly.

The external boundary r_∞ of the integration region should be selected so that, on one hand, the desired functions would fit well with their asymptotic values in the incident flow, while on the other hand, r_∞ should not be too large because otherwise, for a given number of nodal points on the layer, the accuracy in determining r will be poorer. Correctness of selection of values of r_∞ was /80 checked by the fact that the distance between the boundary and the body was increased by a factor of 2 and the problem was solved again with the same value of l and with twice the number of points k. In the general case the value of r_∞ was a function of M_∞, Re, Pr and γ. As a rule, for calculations to three significant figures it is sufficient to take 20 nodal points along r.

After selecting the maximum interval τ (which was found to be of the order of ν), which permits stable calculations for $\varepsilon = \nu$, it is necessary to reduce ε further until oscillations appear near the steady-state solution. The calculation with the smallest value of ε for which these oscillations do not appear is assumed as final. The exactness of solution was checked by comparing solutions obtained with intervals l and $l/2$ for the same values of r_∞ and ε. Some results of numerical calculations are shown in Figs. 2 and 3.

Figure 2a shows the steady-state as a function of ε. It can be seen how, with a decrease in ε, the graph of ρ is smoothed out less, and that fluctuations appear on further reduction [in ε]. Figure 2b presents graphs of u for three calculations: $r_\infty = 1.42$ ($k = 20$), $r_\infty = 1.60$ ($k = 20$) and $r_\infty = 1.84$ ($k = 40$). For $r_\infty = 1.84$ velocity u becomes equal to its asymptotic value in the incident flow practically for $r = 1.55$ and for $r_\infty = 1.42$ function u does not approach the asymptote at all. The value $r = 1.60$ can be regarded as the outside boundary of the integration region. Figure 3 illustrates the "development" of $\rho_u(t)$ for different $\zeta = Re_\infty/Re$.

The final steady solutions are depicted in graphs of Figs. 4-9. Figures 4 and 5 give functions u and ρ for all the calculated Re. These graphs describe the structure of the shock layer and of the shock wave. It can be seen how the shock wave zone is gradually thinned out with an increase in Re. Figures 6-9 depict u, v, p and p_2 as functions of r. For Re = 50 reference should not be made to a shock wave as such, when Re = 100 the shock wave is highly washed /81 out and blends with the boundary layer at the body, when Re = 500 it is already possible to discern a region of inviscid flow inside the shock layer, while for Re = 1000 the inviscid zone comprises a large part of the shock layer (on the graphs u and v in this zone are almost linear). The circles in Figs. 7 and 9 denote solutions obtained by Levinsky and Yoshihara. The qualitative agreement is satisfactory, while the slight deviation may well be attributed to the dif- /82 ference in corresponding values of Re_∞. A sharp increase in the density near the body is observed when Re = 50 (Fig. 5). Possibly this may be attributable to the fact that we have not considered slip at such a low Reynolds number.

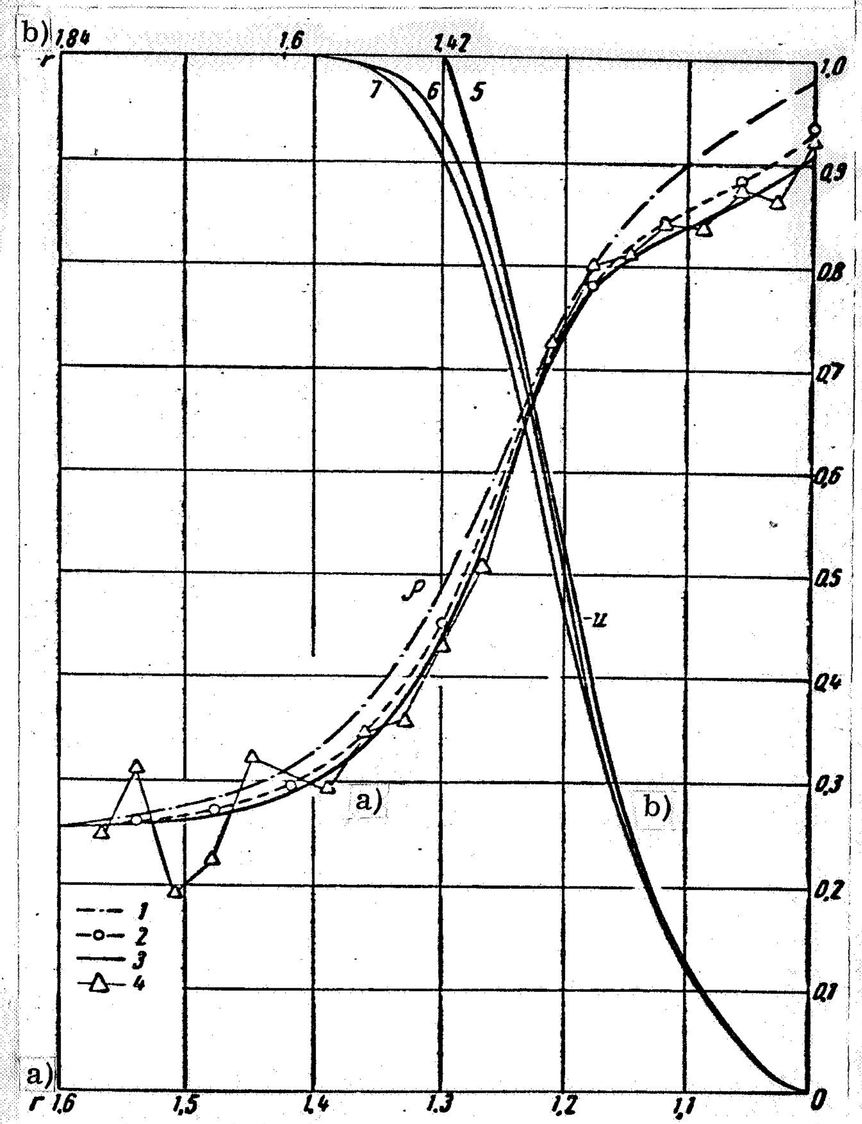

Figure 2. a) The Steady Solution as a Function of Parameter $\varepsilon = \delta l^2/\tau$ (Re = 100, ζ = 1.52, r_∞ = 1.60): 1 - ε = = 0.009, 2 - ε = 0.0036, 3 - ε = 0.0009, 4 - ε = 0.00018. b) Effect of Parameter r_∞ on the steady solution. 5 - r_∞ = = 1.42 (K = 20). 6 - r_∞ = 1.60 (K = 20). 7 - r_∞ = 1.84 (K = 40).

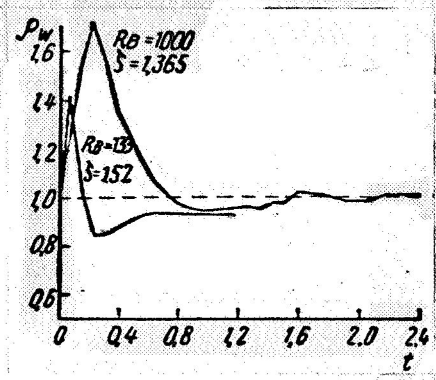

Figure 3. Character of Setting of $\rho_u(t)$.

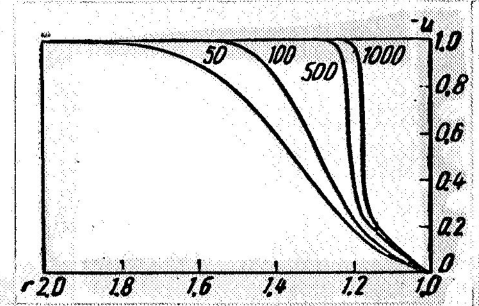

Figure 4. Distributions of Velocity u for Different Reynolds Numbers.

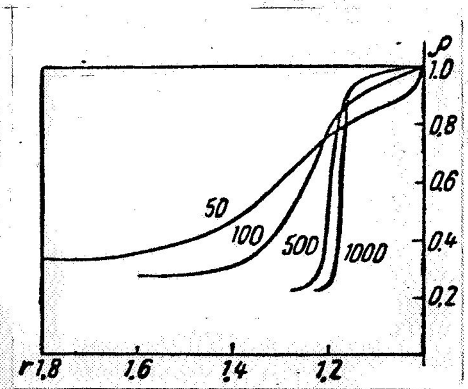

Figure 5. Distributions of Density ρ for Different Reynolds Numbers.

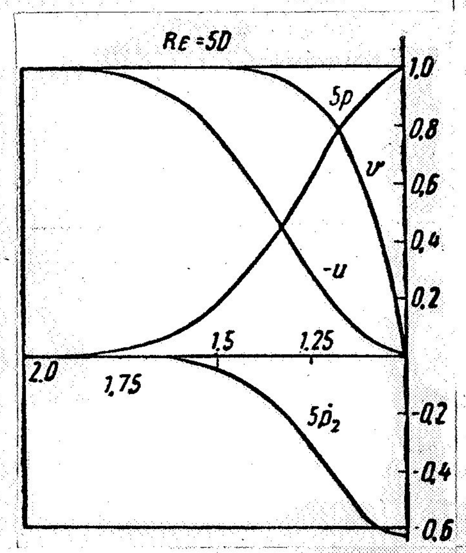

Figure 6. Flow Variables for Re = 50 ($Re_\infty = 95$, $r_\infty = 2.0$: $\tau = 0.02$, $K = 20$, $\delta = 0.02$, $n = 800$).

Thus, we have obtained a singular algorithm for numerically solving the problem which requires no coupling or additional assumptions, which makes it possible to perform calculations for the entire shock layer with gradual transition from the undisturbed incoming flow to the body through the shock-wave zone. This method is suitable also for calculations for plane as well as axisymmetric bodies with nose sections of nearly cylindrical or spherical shape and with thermally insulated or cool surfaces (in this case the energy equation must be used instead of Becker's integral).

I wish to express my thanks to Z. M. Yemel'yanova, who participated in this work.

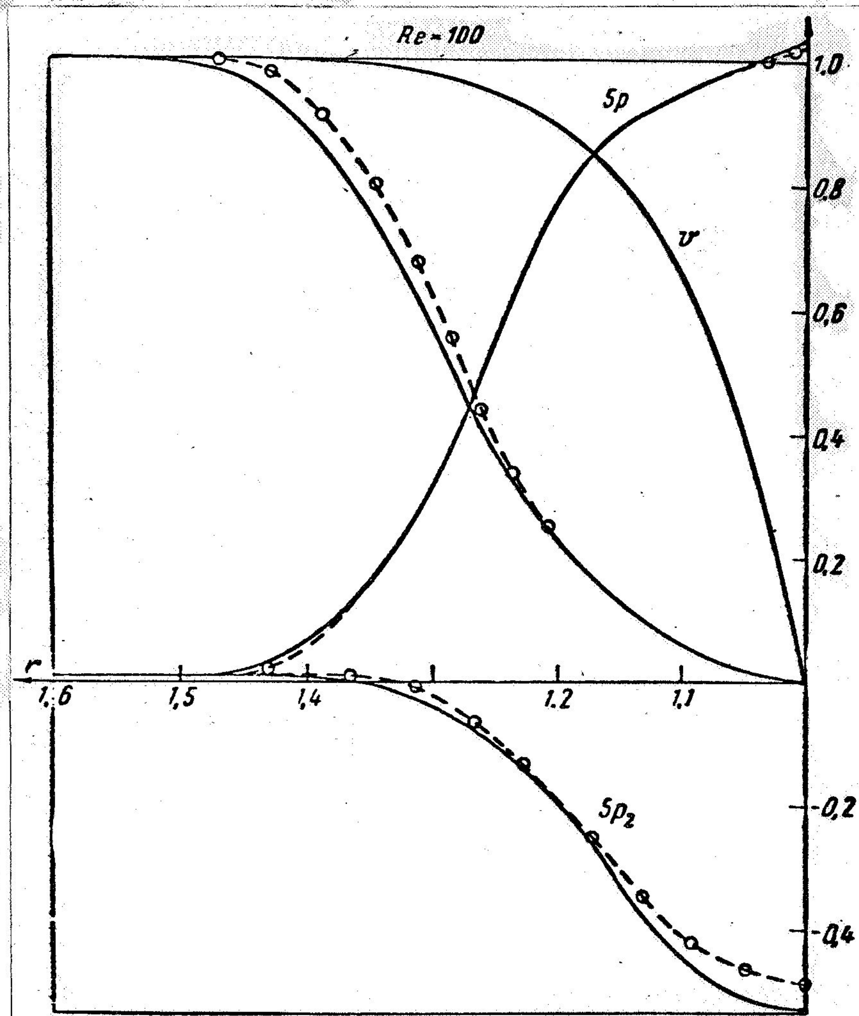

Figure 7. Flow Variables for Re = 100 (Re_{∞} = 164, r_{∞} = =1.60: τ = 0.01, K = 20; δ = 0.01, n = 1000). /79

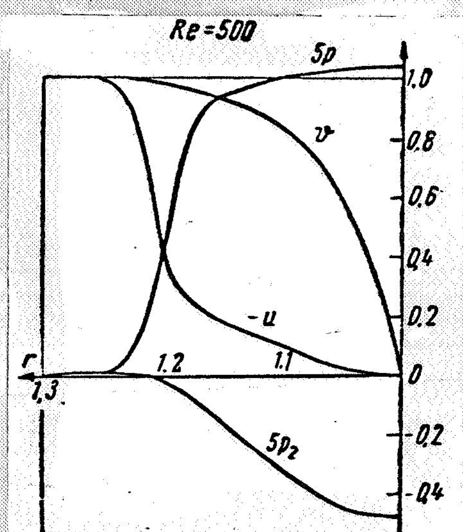

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Figure 8. Flow Variables for Re = 500 (Re_{∞} = 700, r_{∞} = 1.30: τ = 0.001, K = = 20, δ = 0.006, n = 2800).

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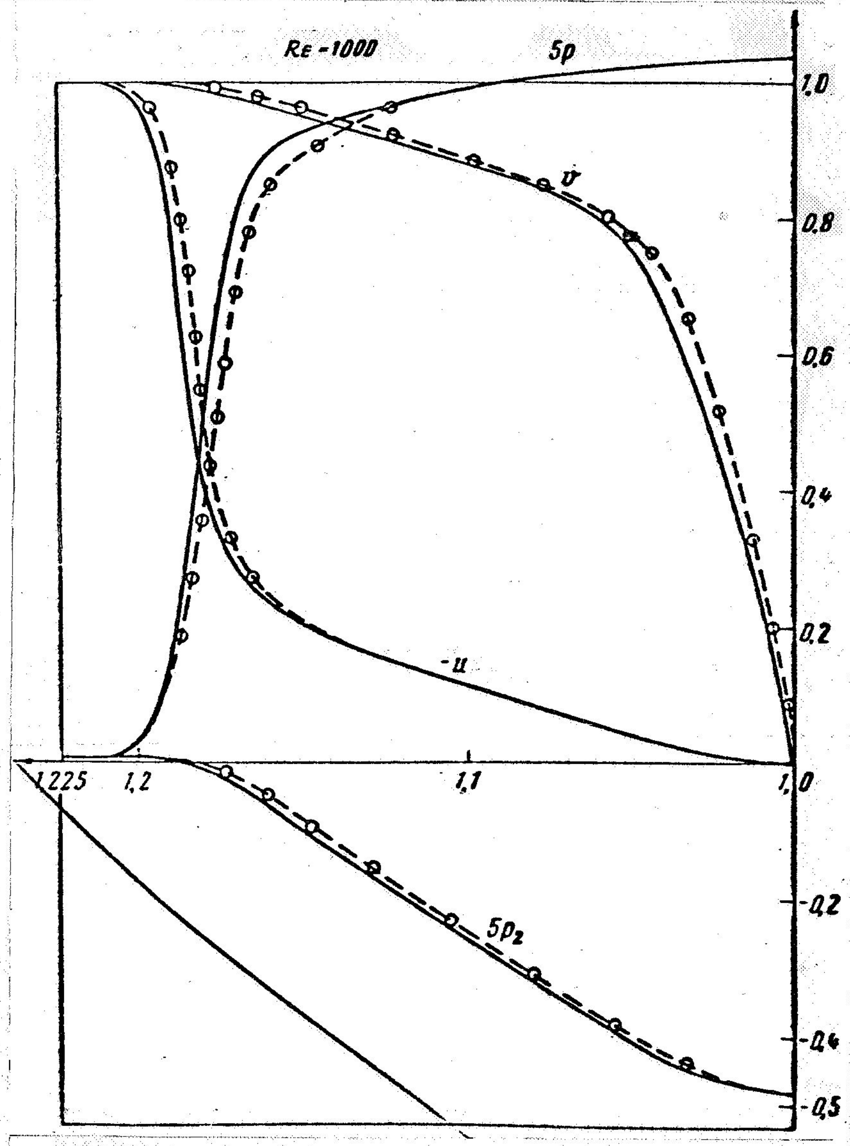

Figure 9. Flow Variables for Re = 1000 (Re_∞=1365, r_∞=1.225: τ=0.0005, k=40, δ=0.0045, n=5200)

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Translated for the National Aeronautics and Space Administration by Scripta Technica, Inc., NASw-1694.